# Status, cognitive overload, and incomplete information in advice-seeking networks: An agent-based model 

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#### Abstract

Advice-seeking typically occurs across organizational boundaries through informal connections. By using Stochastic Actor-Oriented Models (SAOM), previous research has tried to identify the micro-level mechanisms behind these informal connections. Unfortunately, these models assume perfect network information, require agents to perform too cognitively demanding decisions, and do not account for threshold-based critical events, such as simultaneous tie changes. In the context of knowledge-intensive organizations, the shortage of highskilled professionals could determine complex network effects given that many less-skilled professionals would seek advice from a few easily overloaded, selective high-skilled, who are also sensitive to status demotion. To capture these context-specific organizational features, we have elaborated on SAOM with an agent-based model that assumes local information, status-based tie selection, and simultaneous re-direction of multiple ties. By fitting our simulated networks to Lazega's advice network used in previous research, we reproduced the same set of macro-level network metrics with a parsimonious model based on more empirically plausible assumptions than previous research. Our findings show the advantage of exploring multiple generative paths of network formation with different models.


## 1. Introduction

Understanding the formation of social networks requires to consider the context-specific micro-level interplay of network dynamics processes and individual preferences. While there has been progress in the statistical modeling of network data, we still need models that adequately reproduce observed networks by considering contextdependent behavioral assumptions on tie formation (Block et al., 2019; Stadfeld and Amati, 2021).

In knowledge-intensive organizations, research has shown that advice-seeking networks are affected by status-based selection of advisors, mostly related to resources such as skills and expertise (Blau, 1955; McGrath et al., 2003; Lazega et al., 2006; Agneessens and Wittek, 2012; Agneessens et al., 2022). While high-skilled professionals tend to refrain from requesting advice from lower-skilled colleagues to avoid status loss, lower-skilled professionals are keen to receive any advice, preferably from high-skilled advisors (Borgatti and Cross, 2003; Blau, 1955, 1964; Lazega et al., 2012; Agneessens et al., 2022).

This can generate highly centralized networks with high-skilled professionals disproportionately targeted by advice requests (Lazega et al., 2012, 2011), causing their 'cognitive overload', i.e. the failure to cope with an excessive amount of requests (Cross and Prusak, 2002;

Cross et al., 2016; Lazega et al., 2006; Lazega, 2014). Frustrated adviceseekers would therefore reconsider their relationships and target new advisors, thus determining cascading network changes (Granovetter, 1978), with a potentially high amount of ties simultaneously relocated. Research is still needed to test the effect of such micro-macro feedback dynamics on the formation of advice networks.

Previous research has proposed the use of Stochastic Actor-Oriented Models (SAOM; Snijders 2017) to examine the emergence of macrolevel network properties from empirically-estimated micro-level mechanisms (Snijders and Steglich, 2015). Unfortunately, SAOMs make it difficult to examine threshold-based critical events implying simultaneous tie changes, as these models do not allow multiple agents to change their personal relationships at a time (Snijders, 2001). Moreover, there are two key SAOM assumptions that do not reflect certain context-specific cognitive heuristics typical of knowledge-intensive organizations. First, most SAOMs usually assume that agents have full information about other agents' ties (An et al., 2022; Steglich and Snijders, 2022). Instead, in knowledge-intensive organizations, it is not easy for individuals to develop fully accurate perceptions of others' relationships (e.g., Krackhardt 1990, Casciaro et al. 1999, Casciaro 1998) or such information can be strategically concealed (Burt, 1992).

[^0]Second, SAOMs imply that agents evaluate any potential advisor by performing complex, time-consuming calculations, which are incompatible with the heuristic-based decision-making that characterizes empirical agent behavior (Cross et al., 2001b; Cross and Borgatti, 2004).

Here, we propose an agent-based model (Bianchi and Squazzoni, 2015; Macy and Willer, 2002) of the formation of an advice-seeking network that considers these critical aspects. We assumed heterogeneously skilled professionals selecting their advisors according to status-based preferences, and able to re-address their ties in case of cognitive overload of their advisors. As in SAOM, we assumed that agents decide over the state of their outgoing ties by optimizing a set of preferences. However, unlike SAOM, we assumed that agents can make simultaneous, multiple tie changes, have limited information about others' ties, and tend to follow parsimonious behavioral heuristics of advisor selection (Hertwig and Herzog, 2009; Macy and Willer, 2002). Following Snijders and Steglich (2015), we assessed the fit of our model against Lazega (2001)'s classic dataset of advice exchange among lawyers by using a generative approach (Epstein, 2006). We started from a baseline model, and added behavioral assumptions gradually until we generated simulated networks that adequately reproduced the target data. To this end, we performed an exhaustive grid-search of theoretically and empirically plausible parameter combinations.

The remainder of the article is structured as follows. In Section 2, we briefly review how network formation is addressed in SAOMs, with a special focus on their assumptions. In Section 3, we present the model and our simulation results. Section 4 summarizes our main findings and discusses limitations and conclusions.

## 2. Background: SAOMs as agent-based models

### 2.1. Network formation

In SAOM, networks emerge from agents sequentially modifying their own personal relationships (Snijders et al., 2010; Snijders, 2017). Agents follow an objective function that accounts for the change in the state of each of their outgoing tie variables by comparing the expected attractiveness of their own personal networks resulting from each possible decision. This function is a linear combination of count statistics of different resulting network configurations, representing micro-generative processes such as reciprocity, transitivity or popularity, weighted by real-valued parameters, representing the relative magnitude of $i$ 's preferences over such configurations. The objective function for agent $i$ is:
$f_{i}(\boldsymbol{\beta}, X)=\sum_{k} \beta_{k} s_{i k}(X)$,
where $s_{i k}(X)$ represents the count of $k$ th network configuration, calculated on the resulting network $X$ from $i$ 's perspective, while the associated preferences are expresses by $\beta_{k}$, typically estimated from data through the Method of Moments (MoM) (Snijders, 2001).

Because of their agent-based structure and empirical adherence, SAOM can generate networks with realistic macro properties (e.g. connectivity, clustering, etc.) from combinations of micro-generative processes encoded in $s_{i k}(X)$, thus supporting mechanism-based explanations of social networks (Snijders and Steglich, 2015; Stadfeld and Amati, 2021; Steglich and Snijders, 2022).

### 2.2. Assumptions and limitations

In SAOM, any hypothesis about micro-generative processes of network formation is tested by estimating agent preferences from data, and assessing the fit of simulated networks from these parameters with empirical networks, under specific metrics. As long as the fit is not satisfactory, the estimation process is repeated by including new configurations $s_{i k}$, potentially increasing the complexity of the generative mechanism.

Snijders and Steglich (2015) applied this model selection procedure to explain macro-level properties of the advice network sampled by Lazega (2001). Their best-fit objective function included 13 parameters, 12 of which were estimated, while 1 was fixed, representing preferences towards various micro-generative processes. More specifically, agents: (i) had different tendencies to seek and be consulted for advice depending on their seniority; (ii) could evaluate each other's popularity; (iii) reciprocated incoming requests; (iv) considered cyclical advice-exchange patterns; (v) turned to advisors' advisors, modeled as a marginally decreasing function of the number of intermediate edgewise shared partners (GWESP; Hunter 2007); and (vi) were also driven by seniority-based homophily.

However, Snijders and Steglich (2015)'s approach depends on certain behavioral assumptions that - in our opinion - do not adequately reflect the specificity of knowledge-intensive organizational contexts. Indeed, whenever evaluating a potential advisor, agents have full information about the entire state of the network (Steglich and Snijders, 2022; An et al., 2022), i.e. for any number of agents $N$, they are assumed to know the advice activity of colleagues at distance $\geq 2$. For instance, to reproduce observed features of the in- and out-degree distributions of Lazega (2001), Snijders and Steglich (2015) assumed that agents would consider the popularity of potential partners. Seeking a popular partner $j$ is cognitively demanding because $i$ is required to know $j$ 's ties to other parties $h_{1}, h_{2}, \ldots$, before any prior contact with $j$ or with an intermediary agent $k$, which in reality could be unknown. Here, previous research showed that professionals often misperceive or conceal advice activity among each other (e.g., Krackhardt 1990, Casciaro et al. 1999, Casciaro 1998, Burt 1992).

Furthermore, for any adequate empirical specification, SAOMs typically require agents to evaluate potential partners through several cognitively demanding calculations, which in turn involve higher-order network configurations (e.g., GWESP) or increasingly introduce nonlocal informational requirements. For instance, to model hierarchy, Snijders and Steglich (2015) further assumed that agents knew whether potential partners had senior advisors or not. We believe that, in uncertain and knowledge-intensive contexts, it is more plausible to assume that agents follow parsimonious behavioral heuristics (Borgatti and Cross, 2003; Hertwig and Herzog, 2009; Simon, 1956; Carlebach and Yeoung, 2023), speeding-up information retrieval and consumption (Cross et al., 2001b; Cross and Borgatti, 2004).

Finally, in SAOMs advice networks would emerge from sequential agents decisions following a continuous time Markov-Chain process without the possibility to implement simultaneous or cascading changes (Snijders, 2001; Snijders et al., 2010). Following SelfOrganized Criticality theory (SOC; Bak et al. 1987), it is probable that complex organizational settings could spontaneously enter in an overcapacity or saturation state of marginal stability, called the critical state, prompting an immediate re-arrangement of the system itself (Turcotte, 1999). Due to the combination of resources-driven selection of partners, and the typical uneven distribution of resources (Krackhardt, 2003; van der Vegt et al., 2006), high-skilled advisors could be saturated by too many advice requests (Krackhardt, 2003; Lazega et al., 2006; Cross and Prusak, 2002). This could prompt their advice-seekers to look for new, surrogate partners (Lazega, 2014), thereby triggering cascading re-directions of ties which shape the overall network structure. Neglecting these complex dynamics is a serious limitation to our understanding of network formation in knowledge-intensive organizations.

## 3. An ABM of advice network formation

### 3.1. Advice-seeking as a social exchange

An advice relationship can be conceived as a social exchange between seekers and givers (Blau, 1964; Reagans and McEvily, 2003; Lazega et al., 2006), through which seekers aim to obtain valuable
resources such as good-quality information, knowledge or mentoring (Cross et al., 2001a). However, requesting advice generally implies a cost in terms of a status loss (Blau, 1955; McGrath et al., 2003; Agneessens and Wittek, 2012), in that it exposes one's ignorance (Lee, 2002; Borgatti and Cross, 2003), so causing substantial discomfort, which might even be as costly as harming one's reputation (McGrath et al., 2003). Advice relationships occur because the seeker recognizes the expert authority of the giver to compensate for the time dedicated to information provision (Lazega et al., 2006).

Professional skills determine resource endowments sought by advice-seekers (McGrath et al., 2003). In organizational settings, skills tend to be unevenly distributed (Borgatti and Cross, 2003; Krackhardt, 2003; van der Vegt et al., 2006) with a few high-skilled and a majority of less-skilled, more needy professionals (Blau, 1955; van der Vegt et al., 2006).

This led us to assume that agents would possess distinct behavioral rules to seek for advice depending on their own skills (Blau, 1964; Thye, 2000). A minority of high-skilled professionals ( $\alpha$, computed as a percentage of $N$, the overall number of agents) would be primarily sensitive to status loss, caused by making requests to less-skilled professionals. They would prefer avoiding requesting advice or, if any, would prefer to target other high-skilled agents, thus minimizing status loss. On the contrary, low-skilled professionals would seek advice from high-skilled peers, who have better resources (Cross et al., 2001a).

Similarly to SAOM, we assumed that all agents know about the skills-level of all other agents (Steglich and Snijders, 2022), as also observed in various organizations (Cross and Borgatti, 2004; Mirc and Parker, 2020). We modeled the skills-level of agents as a binary attribute.

### 3.2. Baseline model: Heterogeneous status preferences

By using SAOM notation, we represent differences in agents' skillsbased propensities to make advice requests as differently parameterized objective functions. Accordingly, low-skilled agents (denoted with superscript $l$ ) evaluate potential advisors with:
$f_{i}^{l}(\beta, X)=\beta_{0}^{l} \sum_{j \neq i} x_{i j}+\beta_{\text {attract }}^{l} \sum_{j \neq i} x_{i j} T_{j}+\epsilon$
where $\beta_{0}^{l}$ represents the tendency of agent $i$ to send an advice request to any other agent $j \neq i\left(\sum_{j \neq i} x_{i j}\right.$; out-degree count). $\beta_{\text {attract }}^{l}$ represents the attractiveness that low-skilled agents feel towards high-skilled partners ( $\sum_{j \neq i} x_{i j} T_{j}$; count of requests to high-skilled agents, where $T_{j}$ is an indicator function returning 1 if $j$ is high-skilled, 0 otherwise). $\epsilon$ is a disturbance following a Gumbel distribution (Snijders, 2001, 2017).

To consider the reliance of low-skilled agents on high-skilled peers for valuable information, when exploring the parameter space (for detail, see Discussions and Appendix A), we assumed $\beta_{0}^{l}$ to be negative and $\beta_{\text {attract }}^{l}$ to be positive, with $\beta_{\text {attract }}^{l}+\beta_{0}^{l}>0$. For negative $\beta_{0}^{l}$, lowskilled agents have no incentive to send advice requests to low-skilled agents, as this would decrease the outcome of the objective function. For positive $\beta_{\text {attract }}^{l}$, with $\beta_{\text {attract }}^{l}+\beta_{0}^{l}>0$, sending a request to highskilled agents would always increase the objective function. In short, we focused on combinations of parameters that ordered preferences as follows: asking high-skilled agents $>$ doing nothing $>$ asking low-skilled agents. It is worth noting that because of disturbance $\epsilon$, ties between low-skilled agents may occur purely by chance.

We can express the objective function for high-skilled agents (superscript $h$ ) as follows:
$f_{i}^{h}(\boldsymbol{\beta}, X)=\beta_{0}^{h} \sum_{j \neq i} x_{i j}+\beta_{\text {attract }}^{h} \sum_{j \neq i} x_{i j} T_{j}+\epsilon$.
We required $\beta_{0}^{h}$ to be negative and less than $\beta_{0}^{l}$, so that any tie to low-skilled agents significantly reduces the objective function, thereby reflecting the different sensitivity towards status demotion between high- and low-skilled agents. The incentive to avoid status demotion
is only partially offset by the attractiveness towards other high-skilled agents (i.e., here we required $\beta_{\text {attract }}^{h}>0$ but $\beta_{\text {attract }}^{h}+\beta_{0}^{h}<0$ ). In short, we focused on combinations of parameters that ordered preferences as follows: doing nothing $>$ asking high-skilled agents $>$ asking low-skilled agents.

Algorithm 1 summarizes this baseline model. Although we did not expect any empirically realistic networks in this case, exploring extreme conditions was instrumental to better understand the consequences of our assumptions (Epstein, 2006), as well as to further reduce the number of realistic parameter combinations to explore (Miller, 1998).
Algorithm 1 Network formation under heterogeneous status preferences
Require: $N>0$ (number of agents); $\alpha$ (\% of high-skilled agents); $\beta_{0}^{l}, \beta_{0}^{h}, \beta_{\text {attract }}^{l}, \beta_{\text {attract }}^{h}, \epsilon$ (preferences and disturbance); $T$ (number of iterations)
$t \leftarrow 0$
$G=(N, \emptyset) \quad \triangleright$ Initialize an empty network, with $N$ nodes, agents Determine who is high-skilled from data (if available) or randomly while $t \leq T$ do
$i \leftarrow \operatorname{Rand}(1, N) \quad \triangleright$ Randomly select an agent
if $i$ is low-skilled ( $l$ ) then
Evaluate $f_{i}^{l}(\beta, X)$ for each $j \neq i$ and for the do-nothing case Pick $j$ that maximizes $f_{i}^{l}(\beta, X)$, consider to do-nothing Set $x_{i j}$ to $x_{i j}^{ \pm}$, if best option is to add or remove a link else if $i$ is high-skilled ( $h$ ) then Evaluate $f_{i}^{h}(\beta, X)$ for each $j \neq i$ and for the do-nothing case Pick $j$ that maximizes $f_{i}^{h}(\beta, X)$, consider to do-nothing Set $x_{i j}$ to $x_{i j}^{ \pm}$, if best option is to add or remove a link

## end if

$t \leftarrow t+1$
end while

### 3.3. Simulating networks with the baseline model

We performed systematic comparisons between 500 simulated networks from Algorithm 1 and Lazega (2001)'s empirical network by using a modified set of the macro network metrics selected by Snijders and Steglich (2015). More specifically, we kept 9 out of 10 of the original metrics, and also considered three additional ones. We excluded the least upper boundedness (lubness) because of ambiguities in its definitions available in $R$ sna package (Butts, 2023) and Snijders and Steglich (2015). By applying Butts (2023)'s implementation on Lazega (2001)'s network, we obtained a value of 0.11 for its lubness, while in Snijders and Steglich (2015) the value, on the same network, was 1.

We investigated additional macro properties of simulated networks, such as their community structure, by focusing on the number of communities found by the computationally efficient random walks-based algorithm (walktrap) of Pons and Latapy (2005). We then calculated the modularity score induced by the retrieved partitions. We also calculated the median of the distribution of normalized betweenness centrality scores, which is related to the concept of 'structural holes', highly relevant in organizational settings (Burt, 1992).

We kept metrics representing the connectivity, degree distributions, clustering, and hierarchy of networks. Concerning connectivity, we computed the diameter and G50 (i.e., maximum and median shortest paths), the number of connected components, and the size of the largest component. Concerning degree distributions, we computed the scaled variance (i.e., variance divided by the mean) of both in- and out-degree distributions, as well as their Pearson correlation coefficient. For scaled variance, values larger than 1 indicate skewed distributions. The transitivity index captured clustering (i.e. the number of transitively closed triplets divided by the number of two-paths), while the Krackhardt index captured network hierarchy, by measuring the extent through which directed paths ran in one direction only. These metrics reflect emergent characteristics that cannot be readily derived from the values

Table 1
Baseline model satisfying the constraints of Section 3.2.

| Parameter | Value |
| :--- | :--- |
| Baseline propensity low-skilled $\left(\beta_{0}^{l}\right)$ | -1 |
| Baseline propensity high-skilled $\left(\beta_{0}^{h}\right)$ | -3 |
| Attractiveness towards high-skilled for both types $\left(\beta_{\text {attract }}\right)$ | 2.5 |

of agent preferences (Snijders and Steglich, 2015). Hence, they should be considered as the ideal empirical target to assess the fit of our model.

Starting from empty networks, $N=71$ agents (i.e. the size of Lazega 2001's network) were iteratively selected for $T=\{3500,7100,7500\}$ ticks, yielding on average 50, 100, and 105 opportunities to change ties, respectively. Results of the baseline model are shown only for $T=7500$, as the other two scenarios did not yield qualitatively different results (see Supplementary Material, SM, in Appendix B). In a preliminary exploration of the parameter space (see Appendix A for details), we found that $\alpha=0.20$ - i.e. a population with 14 high-skilled and 57 low-skilled agents - generated the most empirically adequate networks. Therefore, results are presented from this parameter specification.

We examined model realizations with values for $\beta_{0}^{l}, \beta_{0}^{h}, \beta_{\text {attract }}^{l}$ and $\beta_{\text {attract }}^{h}$ satisfying the constraints introduced in Section 3.2. Here, we show results for $\beta_{0}^{l}=-1, \beta_{0}^{h}=-3, \beta_{\text {attract }}^{l}=\beta_{\text {attract }}^{h}=2.5$ (see Table 1).

Table Suppl. 2 of the SM (Appendix B) shows a counterfactual case in which $\beta_{\text {attract }}^{h}+\beta_{0}^{h}>0$, meaning that high-skilled agents had a small preference to send advice requests to other high-skilled agents.

### 3.4. Baseline model results

Table 2 shows summary statistics of the macro metrics computed from the baseline model, compared to values calculated on Lazega (2001)'s network.

As expected, Algorithm 1 did not generate networks sufficiently comparable to Lazega (2001), despite similar densities. The median value of the scaled variance of the in-degree distribution was approximately 8 times the empirical one. The same statistic on the out-degree distribution was roughly half of the empirical case. Degree distributions were almost perfectly anti-correlated (-0.993), while in Lazega (2001) we observed a small positive correlation.

More specifically, here we had 14 high-skilled agents with an average in-degree of 57 each (the number of low-skilled agents) and 57 low-skilled agents with very small, randomly generated average indegree (Fig. 1). These differences inflated the variance of the in-degree distribution, and were determined by status preferences: Whenever possible, low-skilled agents sought advice from high-skilled ( $\beta_{\text {attract }}^{l}+$ $\beta_{0}^{l}>0$ ), thus increasing their in-degree. On the contrary, high-skilled agents tended to avoid sending requests ( $\beta_{\text {attract }}^{h}+\beta_{0}^{h}<0$ ), and so did not contribute to the in-degree of low-skilled agents.

A similar process explains the scaled variance of the out-degree distribution. Here, we had 57 low-skilled agents with an average outdegree of 14 and 14 high-skilled agents with very small, randomly generated average out-degree (Fig. 1).

As a result, degree distributions tended to be (almost) perfectly anti-correlated: agents with the highest in-degree (the high-skilled ones) tended to have the smallest out-degree (the opposite holds for low-skilled).

The transitivity index was very high, reaching almost 1 . In the baseline model, agents were more likely to lay in triadic configurations such as in-2-stars than in 2-paths. The few 2-paths tended to turn into transitive ties mainly as an artifact of the difference in status preferences and randomness.

Simulated networks tended to have the smallest possible G50, a very small diameter (2) and were fully connected in a single component. Modularity was generally 0 , indicating the absence of distinct community structures. Indeed, the median number of communities was 1. Finally, the Krackhardt Index and the median normalized betweenness
centrality revealed a very hierarchical structure, with values around 1 and 0 respectively. Therefore, simulated networks were organized in a core-periphery way (Borgatti and Everett, 1999).

### 3.5. Adding cognitive overload

As suggested by Cross and Prusak (2002) and Cross et al. (2016), when advisors are overloaded by too many requests - as in Table 2 and Fig. 1 - they cannot keep up with their own regular work (Zagenczyk and Murrell, 2009), resulting in time bottlenecks that reduce the quality and timeliness of advice and call for counter-action. To implement this 'cognitive overload' mechanism (Kirsh, 2000; Lazega et al., 2006), we introduced a homogeneous tolerability threshold ( $\tau$ ) constraining high-skilled agents' number of possible incoming advice requests from low-skilled agents.

Inspired by self-organized criticality (Bak et al., 1987; Turcotte, 1999), we assumed that whenever a high-skilled agent receives a request which would make its in-degree exceed $\tau$ (Granovetter, 1978), the system entered a critical state. As in earthquakes, where the accumulated stress is instantaneously released (Olami et al., 1992), here we assumed that some low-skilled advice-seekers released their ties from the overloaded advisor. More specifically, a randomly selected subset of its already existing low-skilled seekers instantaneously decided to re-direct their ties away, starting to seek advice from other low-skilled. This mechanism follows Lazega et al. (2006), in that when information "[...] starts becoming inaccessible or inappropriate (irrelevant, inaccurate, untimely), members tend to turn to other sources of advice and create new 'stars' [...]". In our case, these new stars were other low-skilled agents, thus forming a homophilous social niche (Lazega et al., 2016).

Whenever redirecting, low-skilled seekers weighted two different mechanisms: exploring vs. exploiting relationships with other low-skilled agents. Agents explore by choosing to contact advisors of their own advisors, so obtaining resources that neither the seeker nor its advisor has. Formally, this is represented with the count of transitive triplets among low-skilled agents. Through exploitation, agents request advice from an existing advice seeker. Inspired by satisficing principles of bounded rationality (Simon, 1956), the mechanism assumes an incentive for agents to turn to surrogate or social-niche solutions (Lazega, 2014; Lazega et al., 2016)

This led us to re-write Eq. (2) as follows:

$$
f_{i}^{l}(\boldsymbol{\beta}, X)=\beta_{0}^{l} \sum_{j \neq i} x_{i j}+\mathbb{1}\left\{\text { Indeg }_{h}<\tau\right\}\left(\beta_{\text {attract }}^{l} \sum_{j \neq i} x_{i j} T_{j}\right)+
$$

$$
\begin{equation*}
\left(1-\mathbb{1}\left\{\text { Indeg }_{h}<\tau\right\}\right)\left(\beta_{E L}^{l} \sum_{j \in L} x_{i j} x_{j i}+\beta_{E R}^{l} \sum_{j, k \in L} x_{i j} x_{j k} x_{i k}\right)+\epsilon \tag{4}
\end{equation*}
$$

where $\mathbb{1}\left\{\right.$ Indeg $\left._{h}<\tau\right\}$ is an indicator function returning 1 if the in-degree of high-skilled advice-givers of $i$ is strictly lower than the cognitive overload threshold $\tau, 0$ otherwise. If no high-skilled advice giver is overloaded, the second term is used for evaluation (same as Eq. (2)). When a high-skilled advice giver is overloaded, and $i$ decides to redirect, the third term is used to evaluate new advice givers. $\beta_{E R}^{l}$ and $\beta_{E L}^{l}$ are, respectively, the preferences towards exploratory and exploitative ties: While exploration would induce agents to expand their personal neighborhood, exploitation would instead induce them to stay within it. To ensure a high likelihood of low-to-low tie formation when redirecting, we required both $\beta_{E L}^{l}$ and $\beta_{E R}^{l}$ to be positive, with $\beta_{E L}^{l}+\beta_{E R}^{l}+\beta_{0}^{l}>0$.

The full model is represented by Eqs. (3) and (4), and summarized by Algorithm 2. Here, not only do agents behave according to different parameters, they also perform completely different actions (e.g. high-skilled agents do not redirect and do not follow the mechanisms of exploitation and exploration). This allowed us to introduce more behavioral heterogeneity compared to SAOM.

Table 2
Summary statistics of metrics calculated on simulated and on Lazega (2001)'s networks. In bold: Median simulated and empirical values.

| Variable | Simulated networks |  |  |  |  |  |  |  | Empirical <br> Values |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | Mean | Std. Dev. | Min | Pctl. 25 | Pctl. 50 | Pctl. 75 | Max |  |
| Density | 500 | 0.169 | 0.001 | 0.166 | 0.168 | 0.169 | 0.169 | 0.172 | 0.179 |
| Scaled Variance Indegree | 500 | 44.68 | 0.383 | 43.75 | 44.414 | 44.686 | 44.933 | 45.771 | 5.62 |
| Scaled Variance Outdegree | 500 | 2.551 | 0.089 | 2.241 | 2.489 | 2.556 | 2.611 | 2.775 | 4.10 |
| Correlation Indegree-Outdegree | 500 | -0.993 | 0.001 | -0.996 | -0.994 | -0.993 | -0.992 | -0.988 | 0.14 |
| Transitivity | 500 | 0.978 | 0.006 | 0.913 | 0.975 | 0.978 | 0.981 | 0.99 | 0.44 |
| Diameter | 500 | 2.096 | 0.295 | 2 | 2 | 2 | 2 | 3 | 3 |
| G50 | 500 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 2 |
| No. Components | 500 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| Size of largest component | 500 | 71 | 0 | 71 | 71 | 71 | 71 | 71 | 71 |
| Modularity | 500 | 0.001 | 0.002 | 0 | 0 | 0 | 0 | 0.008 | 0.29 |
| No. Communities | 500 | 1.166 | 0.393 | 1 | 1 | 1 | 1 | 3 | 3 |
| Median BTW centrality | 500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.008 |
| Krackhardt H_Index | 500 | 0.996 | 0.006 | 0.959 | 0.996 | 0.999 | 1 | 1 | 0.16 |

Algorithm 2 Network formation from status preferences and cognitive overload
Require: $N>0$ (number of agents); $\alpha$ (\% of high-skilled agents); $\tau$ (cognitive overload threshold); $\beta_{0}^{l}, \beta_{0}^{h}, \beta_{\text {attract }}^{l}, \beta_{\text {attract }}^{h}, \beta_{E L}^{l}, \beta_{E R}^{l}, \epsilon$ (preferences and disturbance); $T$ (number of iterations)
$t \leftarrow 0$
$G=(N, \emptyset) \quad \triangleright$ Initialize an empty network, with $N$ nodes, agents Determine who is high-skilled from data (if available) or randomly Assign $\tau$ to high-skilled agents
while $t \leq T$ do
$i \leftarrow \operatorname{Rand}(1, N) \quad \triangleright$ Randomly select an agent
if $i$ is low-skilled ( $l$ ) then
Evaluate $f_{i}^{l}(\boldsymbol{\beta}, X)$ for each $j \neq i$ and for the do-nothing case Pick $j$ that maximizes $f_{i}^{l}(\beta, X)$, consider to do-nothing if $j$ is high-skilled and In-Degree $(j)>\tau$ then

Remove and redirect between 1 and $\tau l$-agents asking to $j$
for Every redirecting low-skilled $l$ do
Evaluate low-skilled agents via third term of Eq. (4)
Pick $j$ that maximizes $f_{i}^{l}(\beta, X)$, consider to do-nothing
Set $x_{i j}$ to $x_{i j}^{ \pm}$, if best option is to add or remove a link end for
else
Set $x_{i j}$ to $x_{i j}^{ \pm}$, if best option is to add or remove a link end if
else if $i$ is high-skilled ( $h$ ) then
Evaluate $f_{i}^{h}(\boldsymbol{\beta}, X)$ for each $j \neq i$ and for the do-nothing case
Pick $j$ that maximizes $f_{i}^{h}(\beta, X)$, consider to do-nothing
Set $x_{i j}$ to $x_{i j}^{ \pm}$, if best option is to add or remove a link

## end if

$t \leftarrow t+1$
end while

### 3.6. Simulating networks

We first ran simulations with extreme parameters ( $\tau=1$; see Appendix A) to observe outcomes in case high-skilled agents were minimally available to meet advice requests coming from low-skilled agents. This case was exactly the opposite of the baseline model, where agents were maximally available (no $\tau$ ). For other parameters, we show results with $\beta_{0}^{l}=-1$ and $\beta_{0}^{h}=-3$ as before, and with $\beta_{E L}^{l}=\beta_{E R}^{l}=1.5$ (see Table 3). Here, $\beta_{\text {attract }}^{l}=\beta_{\text {attract }}^{h}=2.3$, because 2.5 generated too dense networks (median: 0.2952). Nevertheless, results also hold for 2.5.

We then explored more fine-grained and plausible combinations of $\tau$ and $\beta$, and ran Algorithm 2 until we reached the density of Lazega (2001)'s network. Here, we report a selection of the best-fitting models

Results are shown for $\beta_{0}^{l}=-0.4, \beta_{0}^{h}=-3$, as regards baseline propensity. For attractiveness, $\beta_{\text {attract }}^{l}=\beta_{\text {attract }}^{h}=2.3$. Concerning exploitation $\left(\beta_{E L}^{l}\right)$ vs. exploration ( $\beta_{E R}^{l}$ ), we considered two different

Table 3
Full model with $\tau=1$ satisfying constraints on parameters.

| Parameter | Value |
| :--- | :--- |
| Baseline propensity low-skilled $\left(\beta_{0}^{l}\right)$ | -1 |
| Baseline propensity high-skilled $\left(\beta_{0}^{h}\right)$ | -3 |
| Attractiveness towards high-skilled for both types $\left(\beta_{\text {atract }}\right)$ | 2.3 |
| Preferences towards exploitation $\left(\beta_{E L}^{l}\right)$ | 1.5 |
| Preferences towards exploration $\left(\beta_{E R}^{l}\right)$ | 1.5 |

Table 4
Parameters of good-fitting models.

| Parameter | Value |
| :--- | :--- |
| Both cases |  |
| Baseline propensity low-skilled $\left(\beta_{0}^{l}\right)$ | -0.4 |
| Baseline propensity high-skilled $\left(\beta_{0}^{h}\right)$ | -3 |
| Attractiveness towards high-skilled for both types $\left(\beta_{\text {attract }}\right)$ | 2.3 |
| Reported tuple $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)$ for case 1 |  |
| $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)$ | $(0.5,0.25)$ |
| Reported tuple $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)$ for case 2 | $(0.25,0.5)$ |
| $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)$ |  |

cases. In case 1, low-skilled agents preferred to redirect their ties to other low-skilled agents in their neighborhood (exploitation), rather than asking to advisors' advisors (exploration). In case 2, the opposite was true (see Table 4).

As robustness checks, in the SM (Appendix B) we show results with different magnitudes for $\beta_{E L}^{l}$ and $\beta_{E R}^{l}$. For the first case, we report $\beta_{E L}^{l}$ equal to $\{0.4,0.6\}$, and keep $\beta_{E R}^{l}$ to 0.25 . For case 2 , the exact opposite is reported. Finally, for each parameter combination, we drew $\epsilon$ from a Gumbel distribution with location 0 and scale 0.3.

### 3.7. Results

### 3.7.1. Minimum cognitive threshold

Table 5 shows that even Algorithm 2 with $\tau=1$ did not fit Lazega (2001)'s network. However, here we found empirically plausible values for scaled in- and out-degree variance. Furthermore, the two distributions now started to be strongly positively correlated (in some cases, almost perfectly).

Although high-skilled agents were still frequently requested by lowskilled peers, they were now too easily overloaded to engage in any advice. Because of the redirection mechanism, low-skilled agents created new stars (Lazega et al., 2006) by asking advice from other low-skilled. Therefore, our 14 high-skilled agents now had low in- and out-degree values on average, whereas the 57 low-skilled agents had high in- and out-degree values.

Compared to the baseline model (Table 2), we observed a lower but still very high transitivity. In this case, however, the high values were caused by the exploration mechanism.


Fig. 1. Average in- and out-degree of high- and low-skilled agents, generated from parameters of Table 1.

Table 5
Summary statistics of the metrics computed on simulated and Lazega (2001)'s networks. In bold: Median simulated and empirical values.

| Variable | Simulated networks |  |  |  |  |  |  |  | Empirical <br> Values |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | Mean | Std. Dev. | Min | Pctl. 25 | Pctl. 50 | Pctl. 75 | Max |  |
| Densities | 500 | 0.197 | 0.006 | 0.18 | 0.193 | 0.197 | 0.201 | 0.22 | 0.179 |
| Scaled Variance Indegree | 500 | 6.647 | 2.605 | 3.157 | 4.941 | 6.034 | 7.365 | 21.081 | 5.62 |
| Scaled Variance Outdegree | 500 | 4.049 | 0.226 | 3.071 | 3.895 | 4.045 | 4.203 | 4.63 | 4.10 |
| Correlation InOut | 500 | 0.738 | 0.106 | 0.319 | 0.682 | 0.748 | 0.82 | 0.933 | 0.14 |
| Transitivity | 500 | 0.727 | 0.025 | 0.649 | 0.711 | 0.729 | 0.743 | 0.802 | 0.44 |
| Diameter | 500 | 6.74 | 1.224 | 4 | 6 | 7 | 7 | 12 | 3 |
| G50 | 500 | 2.112 | 0.352 | 1 | 2 | 2 | 2 | 3 | 2 |
| No Components | 500 | 3.242 | 1.519 | 1 | 2 | 3 | 4 | 9 | 1 |
| Modularities | 500 | 0.481 | 0.064 | 0.144 | 0.471 | 0.495 | 0.517 | 0.595 | 0.29 |
| No Communities | 500 | 10.804 | 3.53 | 4 | 8 | 10 | 14 | 19 | 3 |
| Median BTW centrality | 500 | 0.003 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 | 0.011 | 0.008 |
| Krackhardt H_Index | 500 | 0.373 | 0.169 | 0.101 | 0.246 | 0.292 | 0.574 | 0.747 | 0.16 |

Emergent advice networks showed a community structure with high modularity (median: 0.495). Furthermore, because high-skilled agents now were unwilling to provide advice, networks began to disconnect (median number of components: 3 ), with a relatively high number of communities detected via random-walks (median: 10). G50 was similar to empirical values, but the diameters were much larger. The median normalized betweenness centrality and Krackhardt index started to approach empirical levels.

In summary, in the two extreme cases with no $\tau$ and $\tau=1$, we found opposite results for many metrics. This suggested the existence of a sweet-spot region for $\tau$ itself in which empirically valid advice networks could be generated.

### 3.7.2. Intermediate cognitive threshold value

Following Snijders and Steglich (2015), we considered our model to adequately fit a metric as long as its empirical value fell within the central $95 \%$ interval of the corresponding implied distribution, calculated from simulations. For case 1, Fig. 2 shows metrics obtained by having $\beta_{E L}^{l}=0.5$ and $\beta_{E R}^{l}=0.25$. Almost all empirical values were well represented in the implied distributions. Lazega (2001) values for scaled variances and the correlation between in- and outdegree distributions, transitivity index and the number of communities were on the medians of implied distributions or close to. Modularity was farther from the corresponding simulated median, although still within the central $90 \%$ interval (which is an even more stringent test of adequacy in Snijders and Steglich 2015). The empirical median
normalized betweenness centrality was on the third quartile of the simulated distribution. Unfortunately, the Krackhardt hierarchy index was excessively high (simulated median: 0.337 , empirical value: 0.16 ). This would suggest that advice requests in case 1 specification still tended predominantly towards high-skilled.

Note that some metrics have implied distribution with 0 or negligible inter-quartile range (IQR). For these metrics, we found that diameters (median: 3, empirical: 3), G50 (median: 2, empirical: 2), number of components (median: 1, empirical: 1) and size of largest component (median: 71, empirical: 71) perfectly fitted Lazega (2001)'s values.

Fig. 3 shows results for case 2 with $\beta_{E R}^{l}=0.5$ and $\beta_{E L}^{l}=0.25$. While the fit for the degree distributions was still perfect, the fit for transitivity was very poor (median: 0.528 , empirical: 0.44 ). This is due to $\beta_{E R}=0.5$, which is greater than $\beta_{0}^{l}$ in absolute terms. However, the empirical modularity and the corresponding number of communities were now perfectly represented by the simulated distributions. This depended on the different ways to increase local clustering between cases. While in case 1 , low-skilled agents occasionally closed transitive triplets, the opposite was true here. Note that the Krackhardt Index was still not well-represented (median: 0.363).

Even in case 2, simulated diameters, G50, number of components, and size of largest component fitted perfectly the empirical values.

In cases 1 and 2, to a significant extent (see SM), simulated networks appropriately fitted the connectivity, degree distributions, median of the normalized betweenness centrality distribution, modularity, and


Fig. 2. Implied distributions of network metrics calculated from 500 simulations when $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)=(0.5,0.25)$, centered on their medians and scaled by their IQR. Empirical values (blue dots) are centered and scaled as well. Values within red-dashed lines (central $95 \%$ interval) indicate adequate fit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 3. Implied distributions of network metrics calculated from 500 simulations when $\left(\beta_{E L}^{l}, \beta_{E R}^{l}\right)=(0.25,0.5)$, centered on their medians and scaled by their IQR. Empirical values (blue dots) are centered and scaled. Values within red-dashed lines (central $95 \%$ interval) indicate adequate fit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
number of communities of Lazega (2001). Transitivity was not realistic in case 2, whereas perfectly fitted in case 1 . Therefore, of the 12 network properties calculated on Lazega (2001)'s network, 11 could be adequately fit by case 1 . In both cases, the hierarchy of the empirical network was not well fitted.

To investigate this, we found that there were no requests from high-skilled to low-skilled agents in simulated networks. This resulted in reachability graphs with lower density than in the empirical case, which affected the calculation of the Krackhardt index. As a feasibility analysis, under case 1 (Fig. 2), we iteratively and randomly reversed the direction of low-to-high ties one-at-a-time, and checked after how many reversals the index was eventually fit. Surprisingly, we found that 4 or 5 reversals were sufficient to adequately reproduce the Krackhardt


Fig. 4. Implied distributions of network metrics calculated from 500 simulations for case 1 when $\beta_{0}^{h}=-1.1, \beta_{\text {attract }}^{h}=0.46$, centered on their medians and scaled by their IQR. Empirical values (blue dots) are centered and scaled as well. Values within red-dashed lines (central 95\% interval) indicate adequate fit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
index, without reducing the fit of the other metrics. Considering that there could have been at most $14 \times 57$ reversals or high-to-low ties with $\alpha=0.20$, we needed only the $0.5 \%$ of this theoretical maximum for an adequate fit. This motivated us to investigate the parameter space around the best-fit case 1 in more detail (see Appendix A).

Fig. 4 shows the fit for case 1 when $\beta_{0}^{h}=-1.1$ instead of -3 and with $\beta_{\text {attract }}^{h}$ corresponding to the $20 \%$ of $\beta_{\text {attract }}^{l}$ (i.e., to 0.46 ). The empirical value for Krackhardt index finally fell within the mid 95\% of the implied distribution (approximately on the 20th percentile). The overall fit for the other metrics was unchanged excluding the scaled variances of the degree distributions, where empirical values were now closer to the third quartile. Nevertheless, they were still in the IQR, thus very well represented. Metrics with negligible IQR had still a perfect fit. In the SM (Appendix B), we reported an illustrative combination of parameters in which the empirical Krackhardt index was on the median of the simulated distribution. However, the fit for the scaled-variance of the out-degree substantially dropped.

Fig. 5 shows results with $\tau=22$, with all the other parameters as in Fig. 4. Certain metrics improved and others were accurately fitted. For some metrics, the empirical observation was on the mode of the implied distribution (e.g. for the number of communities or the transitivity). Metrics with little-to-no IQR were again perfectly fitted. Fig. 6 shows a perfect reproduction of the number of communities, correlation between degree distributions, and transitivity, with modularity improved, and an unchanged fit of scaled variance of degree distributions and other metrics.

To further validate the best-fitting model specification, we calculated the distribution of agents across communities using simulated networks with exactly the same number of communities as in Lazega (2001)'s network (i.e. 3 communities) for appropriate comparison. In Lazega (2001)'s network, the 3 communities included 31, 23 and 17 agents, respectively. In our best-fit case, the median number of agents by community was 31.50 (IQR:27-37), 22 (IQR: 19-27.75), and 16 (IQR:12-20). Furthermore, we qualitatively assessed the distribution of high-skilled agents by community. Fig. 7 shows that high-skilled agents belonged to different communities, often unevenly distributed.


 in this figure legend, the reader is referred to the web version of this article.)


Fig. 6. Implied distributions of network metrics calculated on 500 simulations in case 1 with $\tau=22, \beta_{0}^{h}=-1.1, \beta_{E R}^{l}=0.28, \beta_{\text {atrract }}^{h}=0.35$, centered on their medians and scaled by their IQR. Empirical values (blue dots) are centered and scaled. Values within red-dashed lines (central $95 \%$ interval) indicate adequate fit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 7. High-skilled agents (in red) by community (networks from Fig. 6). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

## 4. Discussion and conclusion

Drawing on Stochastic Actor-Oriented Models (SAOM) (Snijders et al., 2010; Snijders and Steglich, 2015), we built an agent-based model that examined the emergence of advice-seeking networks in
knowledge-intensive organizational contexts. We concentrated on the concurrent effect of skills-based status concerns, and re-direction of advice requests in case of high-skilled advisors' cognitive overload. We aimed to show that a model other than SAOM could reproduce Lazega (2001)'s network properties while relying on more empirically plausible assumptions. More specifically, our agents did not have full network information, and could not perform demanding and time-consuming calculations to select advisors. Furthermore, we relaxed the assumption of perfect sequentiality of individual decisions, thus introducing the possibility of studying the impact of threshold-based critical events in network formation that generate multiple cascading tie changes (e.g., Granovetter 1978).

Our model accurately reproduced 12 macro-level metrics of Lazega (2001)'s network, 9 of which were considered and fitted in the pioneering article of Snijders and Steglich (2015). To achieve this, Snijders and Steglich (2015) assumed a combination of micro-generative processes representing popularity, reciprocity, higher-order transitivity (GWESP), cyclical advice exchange, seniority-based attractiveness and homophily, which required the estimation of 12 parameters, whereas 1 parameter was fixed (GWESP decay parameter). In our case, by assuming preferences towards advisors reflecting simple status considerations and heterogeneity of skills, and considering critical re-direction of ties due to advisors' cognitive overload, we were able to achieve an adequate fit by exploring the values of 8 parameters only.

To find the best-fitting parameters, we exhaustively explored certain parameter combinations that allowed us to incorporate theoretical and empirical knowledge on the context of knowledge-intensive organizations (see Sections 3.2 and 3.5 ; see Appendix A for further details), while assessing the similarity of the networks generated from each parameter combination to the network of Lazega (2001) with respect to the 12 macro-level metrics. The evaluation was based on whether the empirical values fell within the central 95\% interval of the corresponding simulated distributions.

Note that our exploration strategy varied from the typical Method of Moments (MoM) of SAOM. Indeed, because of the existence of a binary skills-attribute and a threshold of cognitive overload for high-skilled agents not directly observed in Lazega (2001)'s network, we could not use MoM to estimate these unobserved mechanisms. More specifically, we did not have any empirical aggregate counts of low-to-high ties or ties to individuals with a certain threshold to be matched to simulated counterparts. Nevertheless, we all know that it is important to examine unobserved mechanisms as they can play an important role in network formation (An et al., 2022).

This said, our model also has certain limitations. First, professionals' skills are modeled as fixed binary attributes, so agents cannot learn over time. In reality, we should expect that some low-skilled agents could become more knowledgeable because of good advice received from
high-skilled agents or secondary information received from similar peers. Possible extensions of our work could follow Prell and Lo's (2016) model, where agents' level of knowledge co-evolved with the network structure. Secondly, information on skills was public, while we could also expect reputation and gossip regulating information exchange (Ellwardt, 2019). Finally, the model was validated only on a cross-sectional advice network. Extensions to longitudinal data would help us to better disentangle cyclical dynamics between overload, learning, and status-based preferences (Lazega et al., 2011).

However, even considering these limitations, we believe that our study has shown the full potential of ABM to study various microgenerative mechanisms of network formation in given empirical contexts, with an integration of theory and empirical inferences that should be further explored in network formation research. Indeed, previous ABMs using a SAOM framework have not fully exploited the possibility of comparing simulation outcomes with empirical data (see, e.g., Prell and Lo 2016, Bianchi et al. 2020, Daza and Kreuger 2021), which is key to connect modelers coming from a 'generative' ABM framework with social network modelers.

In conclusion, our study indicates the importance of exploring multiple generative paths of advice network formation and dynamics (Squazzoni, 2012) and highlights the key role of ABM in promoting methodological diversity and synergy, while avoiding "causal exclusivism" (Manzo, 2022). In complex social environments, it is plausible that the path from micro conditions to macro network patterns is characterized by "multiple realizability" (Sawyer, 2004), e.g. different mechanisms leading to similar outcomes. This complexity requires "many models" micro-macro generative explorations (Page, 2018), and tight integration between behavioral theories and network data.

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## Model code

The Python code and the related documentation for model implementation, including the R script to analyze our simulated dataset, are available here: https://github.com/ceco51/Status-cognitive-overlo ad-and-incomplete-information-ABM/tree/main. To compute network metrics, we used R packages igraph, (Nepusz, 2023) and sna (Butts, 2023).

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data is available here: https://github.com/ceco51/Status-cognitive-overload-and-incomplete-information-ABM/tree/main/Datasets.

## Appendix A

In ABMs , there is no optimal algorithm to explore the parameter space, i.e. the set of all possible parameter combinations (Carrella, 2021). Rather, the appropriateness of a given exploration strategy should be assessed by considering: (i) the model assumptions; (ii) the model purpose (Edmonds et al., 2019); and (iii) the feasible alternatives.

In our case, our model assumptions prevented us from using the Method of Moments commonly used in SAOM (see Discussions section). Our goal was to find parameter combinations that would make the
simulated networks as close as possible to Lazega (2001)'s network. Inspired by Borgonovo et al. (2022), we decided to follow a constrained, iterative and exhaustive search strategy, which effectively led us to a full-factorial design. By using constraints that incorporate theoretical and empirical knowledge about the context of knowledge-intensive organizations, we were able to reduce the number of possible parameter combinations to be tested to a relatively smaller region of the parameter space. We then decided to investigate this region exhaustively, i.e. testing many parameter combinations within a given resolution and adjusting the resolution as needed. This strategy allowed us to obtain a complete view of the input-output mapping in the region of interest, eliminating the need to implement alternative search heuristics.

More specifically, constraints concerned parameters' types and values. For types, $\tau$ had to be a positive integer, $\alpha$ a scalar between 0.015 and 0.50 (i.e. at least guaranteeing 1 high-skilled agent, and such that high-skilled agents were a minority), while $\beta$ (baselines, attractiveness, exploitation and exploration) could simply be real-valued scalars. However, $\boldsymbol{\beta}$ had to satisfy the theoretically and empirically informed restrictions of Sections 3.2 and 3.5. Algorithm 3 summarizes our exploration strategy.

Algorithm 3 Exploration strategy for model's parameter space.
Require: Model $M$; set of constraints $C$; metrics to evaluate output $O$; measure of fit $F$

- Satisfy $C$ and randomly select a sample of parameters from the allowed combinations
- Evaluate which parameter (or combination of parameters) has the greatest impact on the simulated network metrics in terms of $F$
- Explore "degenerate", baseline models to set limits on such influential parameters
while Simulated metrics are not fitted do
- Define a finer-grained search interval of values for each parameter
- Form a grid i.e., a queue consisting of every possible unique combination of parameters from the previously defined intervals
- Run the model for each element of the grid
- Evaluate the fit for each combination in the grid (compare $O$ with F)
end while
In our case, as a first step, we identified $\alpha$ and $\tau$ to have the greatest impact on most variation in the metrics. Subsequently, we selected particular combinations of $\beta$-parameters that satisfied $C$, and examined degenerate, baseline models (no $\tau, \tau=1$ ) to determine reasonable values for $\tau$. We then performed a preliminary exhaustive search on the grid formed by $\alpha$ in $\{0.05,0.10,0.15,0.20,0.25,0.30,0.35\}$ and coarsely varying $\tau$ in $\{10,15,20,25,30\}$. We found that for $\alpha=0.20$ our simulated networks were consistently more similar to Lazega (2001).

We set $\alpha=0.20$, and iteratively searched for more finegrained intervals by varying $\tau$ and $\beta$. Thus, we first varied $\tau$ in $\{10,12,15,17,18,19,20,22,23,25,30\}$ while simultaneously varying $\beta$ parameters in finer intervals (e.g. we varied $\beta_{E R}^{l}$ and $\beta_{E L}^{l}$ by 0.05 from 0.10 to 0.70 ). We identified a sample of best-fits, and found that, on average, almost all metrics were well reproduced, except for the Krackhardt Hierarchy Index. We then performed a finer-grained exhaustive search based on this sample of best-fit parameters (e.g., $\beta_{E L}^{l}$ and $\beta_{E L}^{l}$ were varied in steps of size 0.03 ). We ultimately achieved generative sufficiency for each metric, identified the best fit, and collected a sample of interesting, similar combinations, which we used as a sensitivity analysis.

Under the procedure described in Algorithm 3, different parameter combinations lead to different probabilities of generating simulated networks similar to Lazega (2001)'s network with respect to the 12 macro-level metrics. Consequently, the best-fitting parameter combinations can be interpreted as values that maximize the joint probability of observing the network characteristics of Lazega (2001) within our simulated distributions. In other words, the best-fitting parameters are those that minimize the distance between the simulated networks and Lazega (2001) with respect to the 12 metrics used.

## Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j. socnet.2023.09.001.

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